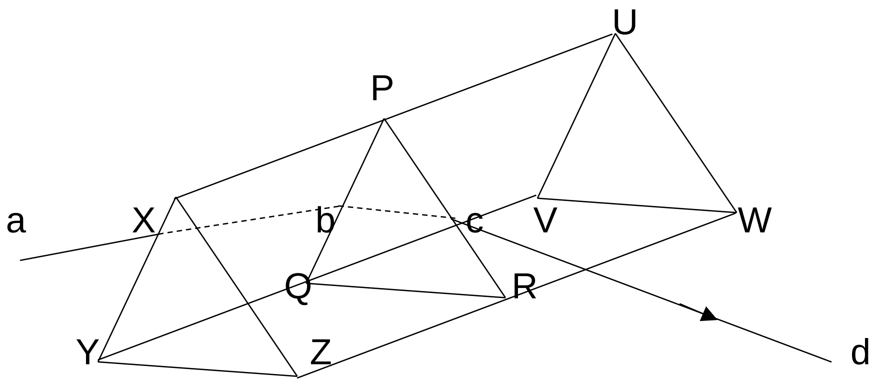
REFRACTION THROUGH PRISMS

A prism is a transparent glass with two plane surfaces inclined to each other.

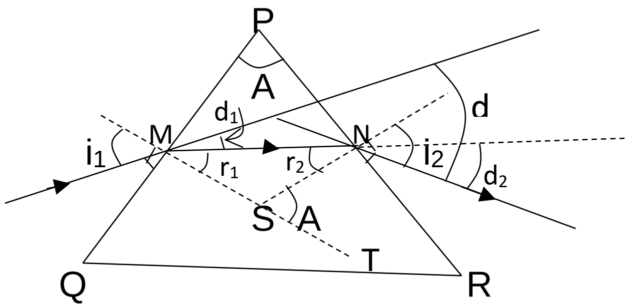


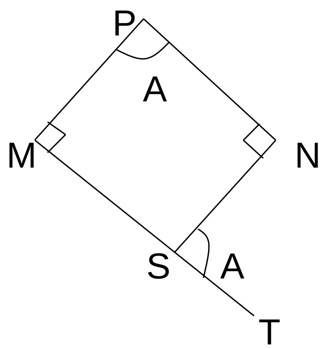
Definitions of terms:

1. Refracting planes: These are the two plane surfaces inclined to each other; that is planes XUVY and XUWZ
2. Refracting edge: This is the line of intersection of the two refracting planes of the prism; that is the line XPU.
3. Principal plane: This is the plane which is perpendicular to the refracting edge, that is planes such as XYZ, PQR and UVW
4. Prism angle: This is the angle between the refracting surfaces of the prism; such as angle YXZ, angle QPR and angle VUW.

DEVIATION OF LIGHT BY A PRISM

It is the change in direction of a ray of light when it passes through a glass prism. The direction of the emergent ray is altered from the initial direction.





The angle through which the beam direction is altered is called deviation, d

the deviation at M;

$$d_1 = i_1 - r_1$$

the deviation at N;

$$d_2 = i_2 - r_2$$

total deviation $d = d_1 + d_2$

$$= (i_1 - r_1) + (i_2 - r_2)$$

$$d = (i_1 + i_2) - (r_1 + r_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

the quadrilateral PMSN cyclic;

$\angle PMS + \angle PNS$

and $\angle MPN + \angle MSN = 180^0$

also $\angle NST + \angle MSN = 180^0$ angles along a straight line

hence $\angle NST = \angle MPN = \angle A$

From triangle NMS; $r_1 + r_2 = A$ …………………… (2)

Minimum deviation D

At minimum deviation occurs the ray of light passes through the prism symmetrically. At minimum deviation the angle of incidence must be equal to the angle of emergence, that is; $i_1 = i_2 = i$ and $r_1 = r_2 = r$

The minimum deviation;

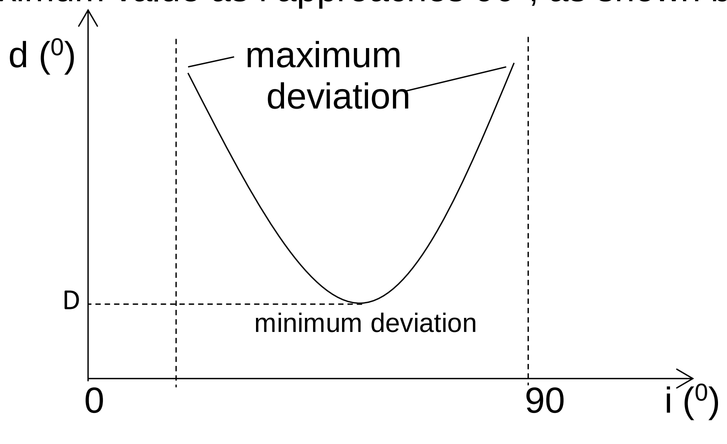
$$D = 2i - 2r$$

From (2) $2r = A$ …………………………………………… (3)

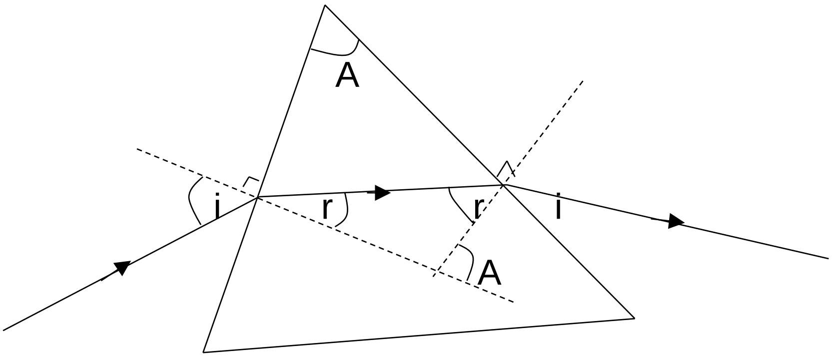
Hence $D = 2i + A$ ……………………………………………. (4)

Variation of angle of deviation with angle of incidence

Experiments show that, as the angle of incidence **i** is increased from zero, the deviation **d** reduces continuously to a minimum value of deviation **D** and then increases to a maximum value as i approaches 90^0, as shown below:



Refractive index at minimum deviation:



Using Snell's law at M

$n \sin i$= constant

$n_a \sin i = n_g \sin r$ ……………………… (5)

From eqn (3)

$2r = A \quad \Rightarrow r = \frac{A}{2}$

And from eqn (4)

$D = 2i - A \quad \Rightarrow i = \left(\frac{A+D}{2}\right)$

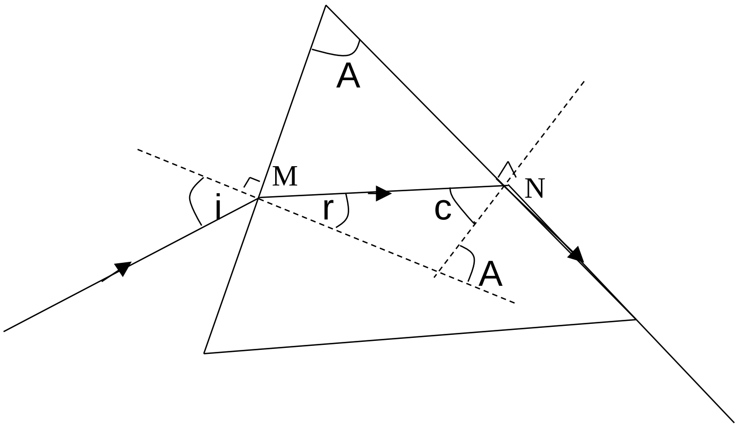
Substituting for i and r in eqn (5)

$\sin\left(\frac{A+D}{2}\right) = n_g \sin(\frac{A}{2})$ since n_a=1

and refractive index $n_g = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin(\frac{A}{2})}$

Grazing emergence (maximum deviation):

For the ray to emerge out of the second refracting surface, at maximum deviation, the angle of emergence is 90^0, that is, the emergence ray grazes the prism surface.



Using Snell's law at M

nsini = constant

sin i = n sin r

but r + C = A ⇒r = A - C

thus sin i = n sin (A-C)

sin i = n sin A cos C –n sin C cos A ……………... (1)

Using Snell's law at N

n sin C = 1x sin 90^0

n sin C = 1

sin C = $\frac{1}{n}$ ……………………. (2)

since $\cos^2$ C + $\sin^2$ C= 1

then cos C = $\sqrt{(1 - sin^2 C)}$ = $\sqrt{\left(1 - \frac{1}{n^2}\right)}$

cos C= $\frac{1}{n}\sqrt{n^2 - 1}$ ……………... (3)

using eqns (2) and (3) into (1)

sin i =n x $\frac{1}{n}\sqrt{n^2 - 1}$ sin A – n x $\frac{1}{n}$ cos A

sin i = $\sqrt{n^2 - 1}$ sin A – cos A

$\sqrt{n^2 - 1}$ sin A = sin i + cos A

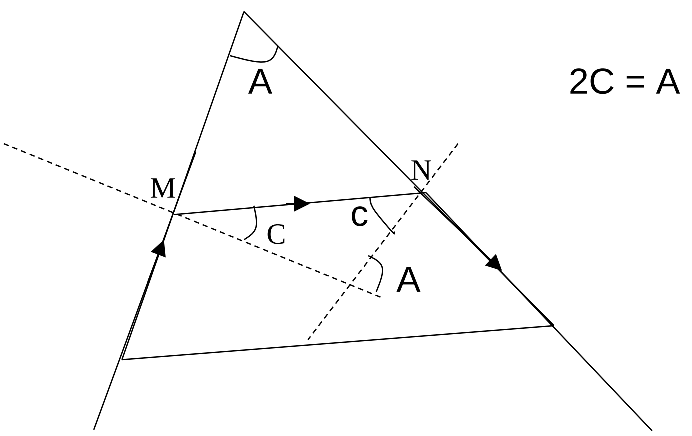
$\sqrt{n^2 - 1}$ = $\frac{\sin i + \cos A}{\sin A}$

n^2 -1 = $\left(\frac{\sin i + \cos A}{\sin A}\right)^2$

hence n = $\sqrt{\left(\frac{\sin i + \cos A}{\sin A}\right)^2 + 1}$

<u>Maximum prim angle (grazing incidence and grazing emergence)</u>:

If the prism angle is increased while maintaining the grazing emergence, the angle of incidence increase until it becomes 90^0, that the incident ray also grazes the refracting surface of the prism.



The largest angle of a prism for which emergent rays are obtained is called the **limiting angle of the prism**. From geometry in the figure above, the limiting angle of the prism is twice the critical angle, that is A= 2C

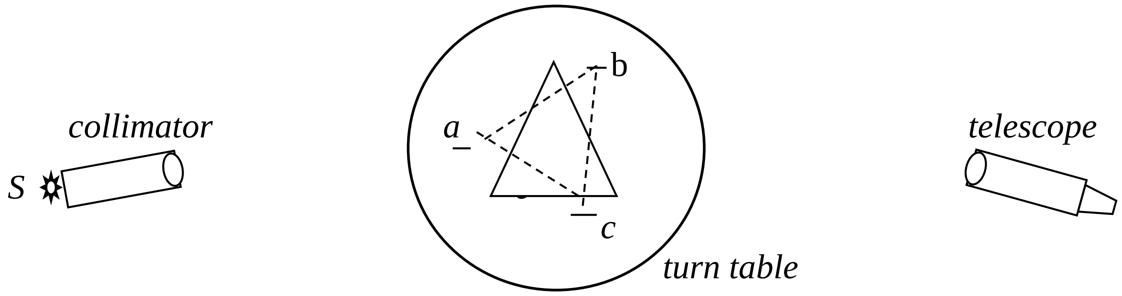
WORKED OUT EXAMPLES (*leave 3 pages space*)

THE SPECTROMETER

This is an optical instrument used to study light from different sources. It consists of three main components namely:-

- Collimator
- Turntable
- Telescope

<u>Structure:</u>



- The collimator produces a parallel beam of light from the monochromatic source of light S
- The turn table has three levelling screws a, b and c; and a circular scale around it, it is where the prism is placed
- The telescope focuses a parallel beam of light emerging out of the prism

<u>Adjustments of the spectrometer</u>:

Before the spectrometer is put in to use, 3 adjustments must be made onto it and these include:-

(i) The collimator is adjusted to produce parallel rays of light.
(ii) The telescope is adjusted to receive light from the collimator on its cross wire.
(iii) The turn table is levelled.

Telescope adjustment:

- The eye piece of the telescope is adjusted so that the cross wires are in sharp focus.
- The telescope is turned to face a distant object. The length of the telescope is adjusted until the image of the image of the distant object is clearly seen on the cross wires. This means that the telescope receives parallel light.

Collimator adjustment:

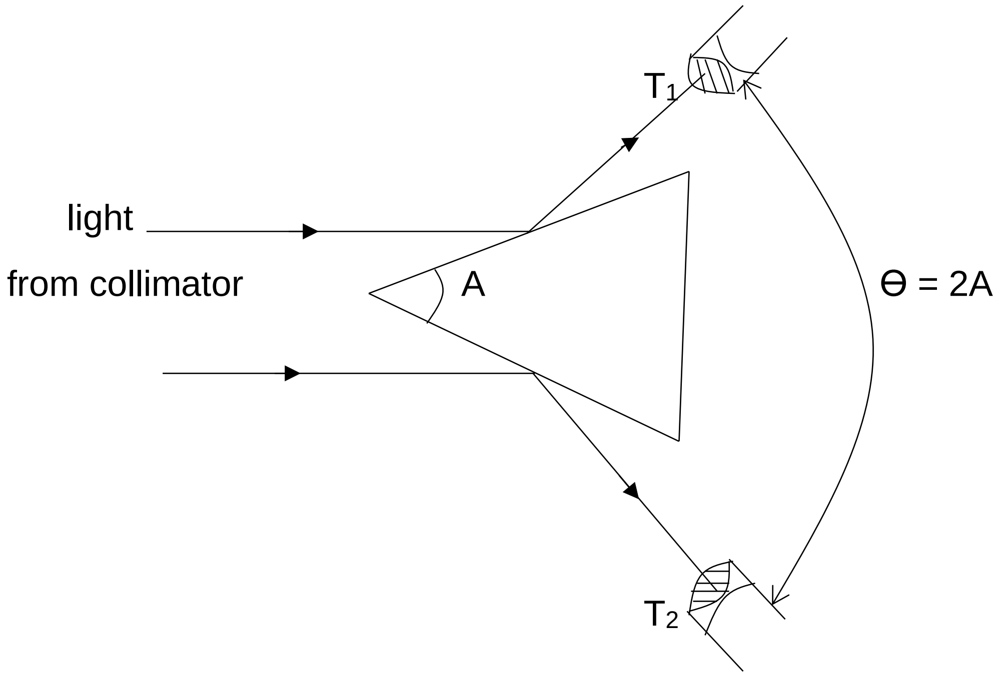
- The prism is removed and the collimator slit is now illuminated using a strong source of monochromatic light.
- The telescope is now turned to face the collimator and collimator length adjusted until the image of its slit is seen clearly on the cross wires. This means that the collimator is set to produce parallel light.

Levelling the turn table

- The prism is now placed on the table with one of its refracting surface perpendicular to the line joining any two levelling screws.
- The prism is turned so that its refracting edge faces the collimator. The light reflected from the refracting surface is observed through the telescope.
- The levelling screws are adjusted in or out to bring the image of the collimator to the centre of the field of view. This way the spectrometer is adjusted and ready for use

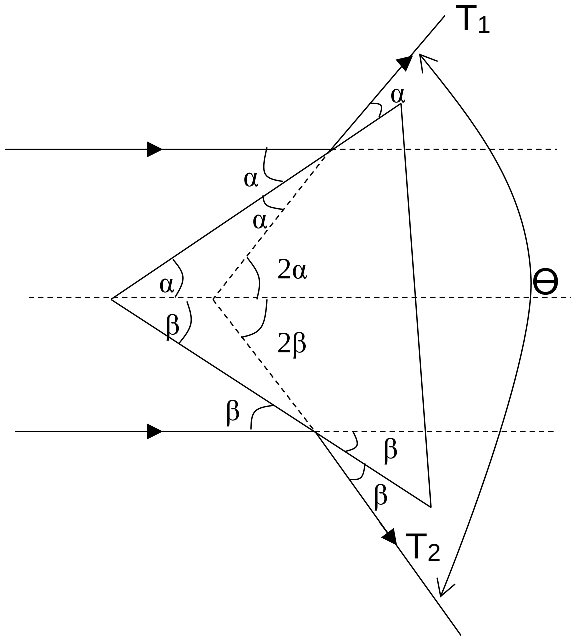
MEASUREMENT OF THE PRISM ANGLE, A

<u>Method 1</u>: Using a spectrometer



- The prism is placed on the turn table with its refracting edge facing the collimator as shown.
- With the table fixed, the telescope is moved to position T_1 to receive the light from the collimator on its cross wire. This position T_1 is noted.
- The telescope is now turned to a new position T_2 to receive light on its cross wire. This position T_2 is also noted.
- The angle Θ between T_1 and T_2 is calculated.
- The prism angle A is determined from the expression $\Theta = 2A$

Theory of experiment:



Let α and β be the glancing angles on the two refracting surfaces respectively,

The reflected rays along T_1 and T_2 respectively are deviated through 2α and 2β respectively.

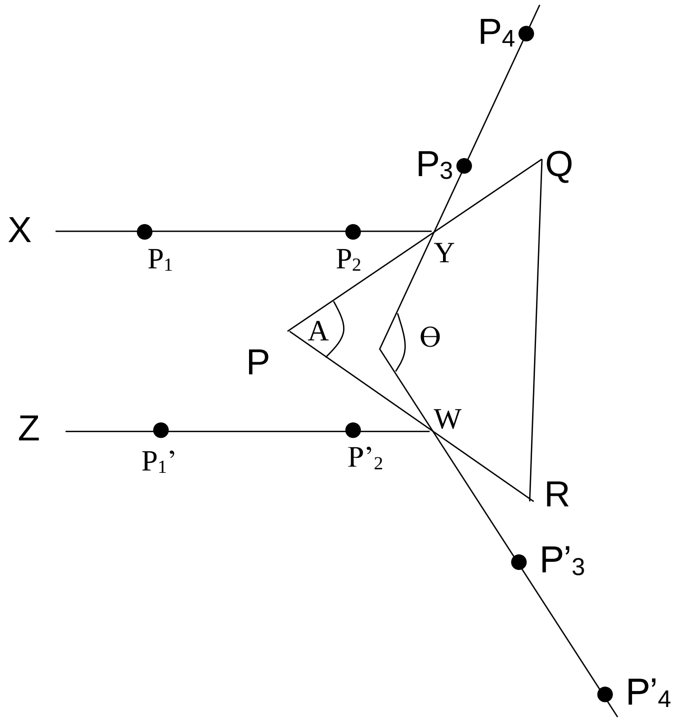
Total deviation $\Theta = 2\alpha + 2\beta$

$\Theta = 2(\alpha+\beta)$

But $\alpha+\beta = A$

Hence $\Theta = 2A$

Method 2: using pins

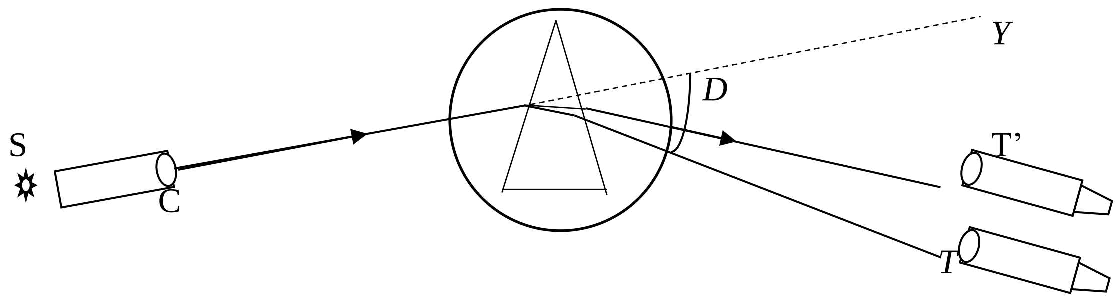


- A white paper is stuck to the soft board using top-headed pins. Two parallel line XY and ZW are drawn on the paper
- The prism is placed with its refracting surfaces touching each line respectively as shown.

- Two optical pins P_1 and P_2 are placed along the line XY. Pins P_3 and P_4 are placed such that they appear to be in line with the images of P_1 and P_2 as seen by reflection from face PQ .
- The procedure is repeated using pins P'_1 and P'_2 along line ZW. Pins P'_3 and P'_4 are placed such that they appear to be in line with the images of P'_1 and P'_2 as seen by reflection from face PR .
- The prism is removed and a line is drawn through positions of pins P_3 and P_4 Another line is drawn through the positions of pins P'_3 and P'_4 . The two line are extended to meet and the angle Ɵ between the two lines is measured and recorded.
- The prism angle A is then calculated from Ɵ = 2A

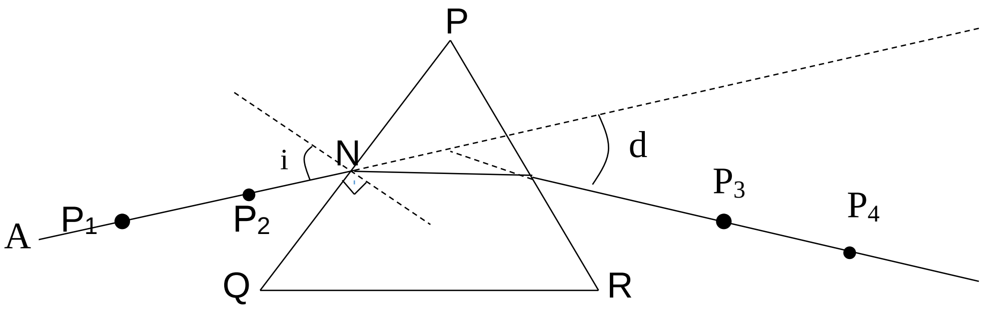
MEASUREMENT OF THE MINIMUM ANGLE OF DEVIATION, D.

<u>Method 1</u>: Using a spectrometer

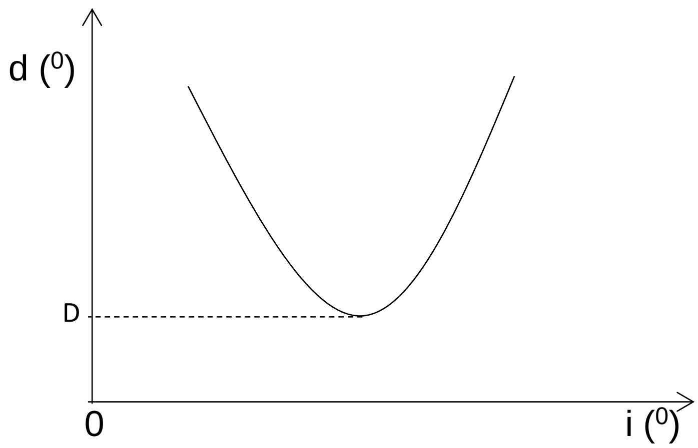


- The prism is placed on the turntable with its refracting angle facing away from the collimator, C as shown in the figure above
- With the telescope T' on the opposite side of the collimator, C the telescope is turned to a position where the image of the collimator slit can be focused.
- The turntable is rotated in order to increase the angle of incidence on the first face of the prism
- Initially the telescope and turntable have to be turned in same direction in order to keep the image of collimator in focus.
- At some position T, of the telescope; it is found that the telescope must be turned in the opposite direction in order to keep the image of collimator in focus as the turntable is turned further.
- This position T, of the telescope is noted from the circular scale.
- The prism is removed from the turntable and the image of collimator is focused directly.
- The new position Y, of the telescope is noted from the circular scale.
- The angle between the positions T and Y of the telescope is determined and this gives the minimum angle deviation, D.

Method 2: using pins



- The prism is placed on a plane sheet of paper on a soft board and its outline PQR is traced out as shown above. The prism is then removed.
- A normal is drawn through point N on side PQ of the prism and a line AN is drawn at an angle of incidence i .
- Two optical pins $P_1$and P_2 are placed along the line AN and prism is placed back on its outline.
- While viewing from the opposite side of the prism; Pins P_3 and P_4 are placed such that they appear to be in line with the images of P_1 and P_2.
- The prism is removed and the angle of deviation d is measured and recorded.
- The procedure is repeated for different values of the angle of incidence i.
- A graph of d against i is plotted and give a curve as shown below.



- The angle of minimum deviation D is obtained from the graph

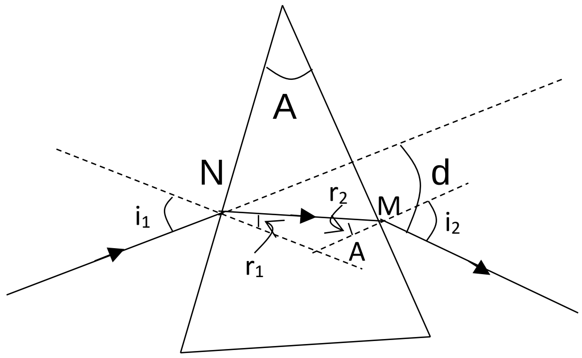
MEASUREMENT OF THE REFRACTIVE INDEX.

- The prism angle A, and the minimum angle of deviation D, are first determined.
- The refractive index, n of the prism is then calculated from the expression $n=\frac{\sin(\frac{A+D}{2})}{sin\frac{A}{2}}$.

DEVIATION OF LIGHT BY A SMALL ANGLE PRISM FOR SMALL ANGLES OF INCIDENCE.

A small angle prism is one whose prism angle is less than 10^0.

Consider a ray incident at a small angle of incidence i_1 and emerges out of the refracting surface at an angle of emergence i_2 as shown in the figure below



Using Snell's law At N; n sin i = constant

$\sin i_1 = n \sin r_1$

for small angles measured in radians; $i_1 \approx \sin i_1$ and $r_1 \approx \sin r_1$

hence $i_1 = nr_1$ .... (1)

Using Snell's law At M;

$n \sin r_2 = \sin i_2$

for small angles measured in radians; $i_2 \approx \sin i_2$ and $r_2 \approx \sin r_2$

hence $nr_2 = i_2$ .... (2)

the total deviation $d = (i_1 + i_2) - (r_1 + r_2)$ ………………………… (3)

substituting for i_1 and i_2 in (3) gives

$d = (nr_1 + nr_2) - (r_1 + r_2)$

$d = n(r_1 + r_2) - (r_1 + r_2)$

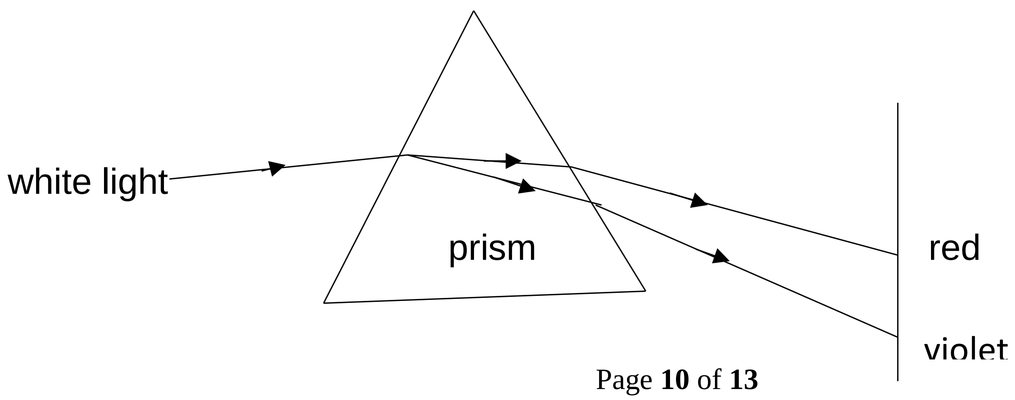
$d = (n-1)(r_1 + r_2)$

but $(r_1 + r_2) = A$

hence $d = A(n-1)$

DISPERSION OF WHITE LIGHT

Dispersion of white light is the separation of white light into its component colours by a transparent medium.

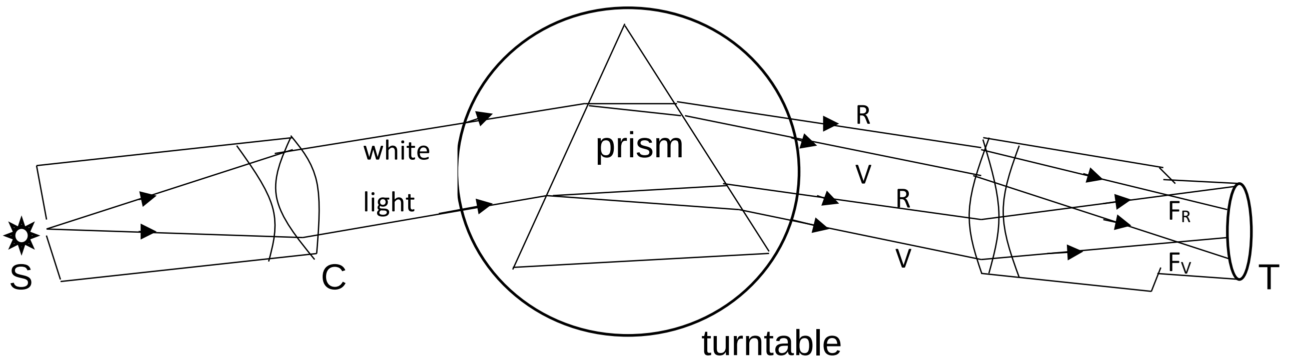


Explanation of Dispersion:

The red light is least deviated while violet is most deviated. Since the angle of incidence in air is the same; the angle of refraction made by red in glass is greater than the angle of refraction for violet then from n = $\frac{\sin i}{\sin r}$ then the refractive index of the prism for red is less than that for violet.

Production of the pure Spectrum

A pure spectrum is one in which the coloured images contain light of one colour only, that is, the images are monochromatic. To produce a pure spectrum for white light, the collimator is made narrow and both the collimator and telescope are adjusted for parallel light.



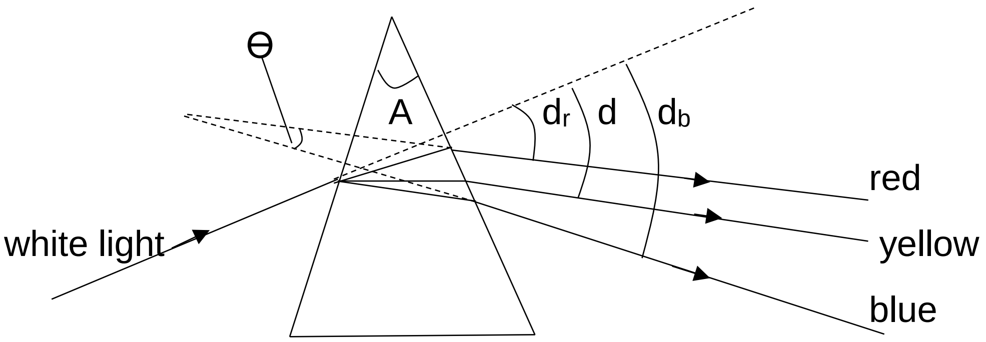
- A bright source of white light S is placed near the collimator slit. The prism is placed on the turntable with one of its refracting surface towards the collimator, C
- The rays refracted through the prism are separated a number of different coloured parallel beams of light each travelling in slightly different directions.
- The telescope T, brings each coloured beam of light to a separate focus such as F_R and F_V.
- A pure spectrum consisting of a series of chromatic images of the slit is observed through the telescope.

Conditions for formation of a pure spectrum:

- The white light must be admitted through a narrow opening in order to reduce chances of overlapping images.
- The beams of coloured light emerging out of the prism must be parallel so that each beam is brought to a separate focus.

Dispersion of light by a small angle prism:

<u>Definitions of terms used</u>:



1) Angular dispersion:

The angular dispersion between red and blue is the angle between the two dispersed rays. This is the same as the difference in deviation for the two colours produced by the prism.

For a small angle prism of refracting angle A and refractive index n;

the deviation for red is given by $d_r = (n_r - 1)A$

while the deviation for blue is given by $d_b = (n_b - 1)A$

the angular dispersion $\Theta = d_b - d_r$

$$\Theta = (n_b - 1)A - (n_r - 1)A = (n_b - n_r)A$$

2) Mean deviation:

This is the deviation of yellow light since yellow is the colour in the middle of the spectrum. For a small angle prism of refracting angle A and refractive index n, the mean deviation is given by $d = (n-1)A$

3) Dispersive power, ω

The dispersive power of the prism for blue and red (any two colours) is the ratio of the angular dispersion to the mean deviation.

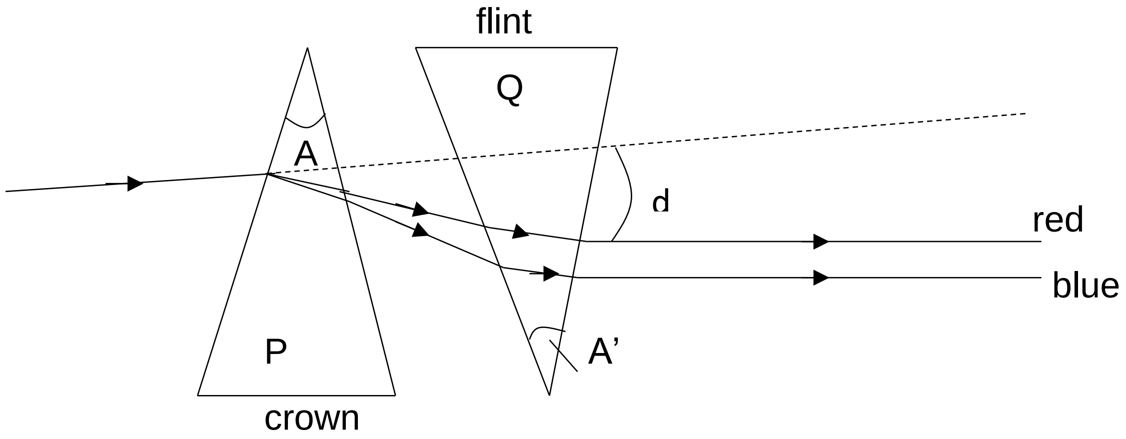
$$\omega = \frac{angular\ dispersion,\ \theta}{mean\ deviation,\ d}$$

$$\omega = \frac{(n_b - n_r)A}{(n-1)A} = \frac{(n_b - n_r)}{(n-1)}$$

EXAMPLES (*leave half a page space*)

ACHROMATIC PRISMS

These are a pair of prisms that can deviate white light without dispersing it into colours.



- Prism P made of crown glass causes dispersion between red and blue in the incident white light. Prism Q made of flint glass is inverted in respect to P; and makes the red and blue incident on it to emerge out of it parallel to each other.
- When the rays are viewed the eye brings them to the same focus on the retina and hence the colour effect due to red and blue is eliminated.

<u>Choice of achromatic prisms</u>

Suppose n_r and n_b are the refractive indices of crown glass for red and blue respectively, then

$$\text{angular dispersion} = (n_b - n_r)A$$

similarly if n'_r and n'_b are the refractive indices of flint glass for red and blue respectively, the angular dispersion = $(n'_b - n'_r)A'$

Prism P produces a downward dispersion while prism Q produces an upward dispersion. For achromatic prisms the angular dispersion produced by P and Q must be equal, that is,

$$(n_b - n_r)A = (n'_b - n'_r)A'$$

<u>Deviation by achromatic Prisms</u>

The angle of deviation d is the angle between the incident and emergent beams.

- The mean deviation by prism P is $d_1 = (n-1)A$ and is in downward direction; while the mean deviation by prism Q is $d_2 = (n'-1)A'$ in the upward direction.
- The net deviation $d = d_1 - d_2$

$$d = (n-1)A - (n'-1)A'$$

EXAMPLES (*leave half a page space*)